Wednesday, October 14, 2015

p. 521: 1, 3, 7, 8, 10, 11, 12, 13, 15, 26

Problem 1

Problem. Identify u and dv for finding the integral $\int xe^{2x}\,dx$ using integration by parts.

Solution. Let $u = x$ and $dv = e^{2x}\,dx$.

Problem 3

Problem. Identify u and dv for finding the integral $\int (\ln x)^2\,dx$ using integration by parts.

Solution. Let $u = (\ln x)^2$ and $dv = dx$.

Problem 7

Problem. Evaluate the integral $\int x^e \ln x\,dx$ using integration by parts with $u = \ln x$ and $dv = x^3\,dx$.

Solution. We have

$$du = \frac{1}{x}\,dx$$

and

$$v = \frac{1}{4}x^4.$$

So

$$\begin{aligned}\int x^e \ln x\,dx &= \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx \\ &= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx \\ &= \frac{1}{4}x^4 \ln x - \frac{1}{4} \cdot \frac{1}{4}x^4 + C \\ &= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C\end{aligned}$$

Problem 8

Problem. Evaluate the integral $\int (4x+7)e^x\,dx$ using integration by parts with $u = 4x+7$ and $dv = e^x\,dx$.

Solution. We have

$$du = 4\,dx$$

and

$$v = e^x.$$

So

$$\begin{aligned}\int (4x+7)e^x\,dx &= (4x+7)e^x - \int 4e^x\,dx \\ &= (4x+7)e^x - 4e^x + C \\ &= (4x+3)e^x + C.\end{aligned}$$

Problem 10

Problem. Evaluate the integral $\int x\cos 4x\,dx$ using integration by parts with $u = x$ and $dv = \cos 4x\,dx$.

Solution. We have

$$du = dx$$

and

$$v = \frac{1}{4}\sin 4x.$$

So

$$\begin{aligned}\int x\cos 4x\,dx &= \frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,dx \\ &= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x + C\end{aligned}$$

Problem 11

Problem. Find the indefinite integral $\int xe^{-4x}\,dx$.

Solution. Let $u = x$ and $dv = e^{-4x}\,dx$. Then $du = dx$ and $v = -\frac{1}{4}e^{-4x}$. So

$$\begin{aligned}\int xe^{-4x}\,dx &= -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x}\,dx \\ &= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C \\ &= -\frac{1}{16}(4x+1)e^{-4x} + C.\end{aligned}$$

Problem 12

Problem. Find the indefinite integral $\int \frac{5x}{e^{2x}}\,dx$.

Solution. Let $u = 5x$ and $dv = e^{-2x}\,dx$. Then $du = 5\,dx$ and $v = -\frac{1}{2}e^{-2x}$. So

$$\begin{aligned}\int \frac{5x}{e^{2x}}\,dx &= -\frac{5}{2}xe^{-2x} + \int \frac{5}{2}e^{-2x}\,dx \\ &= -\frac{5}{2}xe^{-2x} - \frac{5}{4}e^{-2x} + C \\ &= -\frac{5}{4}(2x+1)e^{-2x} + C.\end{aligned}$$

Problem 13

Problem. Find the indefinite integral $\int x^3e^x\,dx$.

Solution. This one is interesting.

Let $u = x^3$ and $dv = e^x\,dx$. Then $du = 3x^2\,dx$ and $v = e^x$.

$$\int x^3e^x\,dx = x^3e^x - \int 3x^2e^x\,dx.$$

Now let $u = 3x^2$ and $dv = e^x\,dx$. Then $du = 6x\,dx$ and $v = e^x$.

$$\begin{aligned}\int x^3e^x\,dx &= x^3e^x - \left(3x^2e^x - \int 6xe^x\,dx\right) \\ &= x^3e^x - 3x^2e^x + \int 6xe^x\,dx.\end{aligned}$$

Finally, let $u = 6x$ and $dv = e^x\ dx$. Then $du = 6\ dx$ and $v = e^x$.

$$\begin{aligned}\int x^3e^x\ dx &= x^3e^x - 3x^2e^x + \left(6xe^x - \int 6e^x\ dx\right)\\ &= x^3e^x - 3x^2e^x + 6xe^x - \int 6e^x\ dx\\ &= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C\\ &= (x^3 - 3x^2 + 6x - 6)e^x + C.\end{aligned}$$

Problem 15

Problem. Find the indefinite integral $\displaystyle\int t\ln(t+1)\ dt$.

Solution. Let $u = \ln(t+1)$ and $dv = t\ dt$. Then $du = \frac{dt}{t+1}$ and $v = \frac{1}{2}t^2$.

$$\begin{aligned}\int t\ln(t+1)\ dt &= \frac{1}{2}t^2\ln(t+1) - \int \frac{t^2}{2(t+1)}\ dt\\ &= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int \frac{t^2}{t+1}\ dt.\end{aligned}$$

To complete the integration, we need to use long division and get

$$\frac{t^2}{t+1} = t - 1 + \frac{1}{t+1}.$$

Now we finish the integration.

$$\begin{aligned}\int t\ln(t+1)\ dt &= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\int\left(t - 1 + \frac{1}{t+1}\right)\ dt\\ &= \frac{1}{2}t^2\ln(t+1) - \frac{1}{2}\left(\frac{1}{2}t^2 - t + \ln|t+1|\right)\\ &= \frac{1}{2}t^2\ln(t+1) - \frac{1}{4}t^2 + \frac{t}{2} - \frac{1}{2}\ln|t+1| + C\\ &= \frac{1}{2}(t^2-1)\ln(t+1) - \frac{1}{4}(t^2 - 2t) + C.\end{aligned}$$

Problem 26

Problem. Find the indefinite integral $\displaystyle\int x^2\cos x\ dx$.

Solution. Let $u = x^2$ and $dv = \cos x\ dx$. Then $du = 2x\ dx$ and $v = \sin x$.

$$\int x^2 \cos x\ dx = x^2 \sin x - 2 \int x \sin x\ dx.$$

To complete the integration, use integration by parts again. Let $u = x$ and $dv = \sin x\ dx$. Then $du = dx$ and $v = -\cos x$.

$$\begin{aligned}
\int x^2 \cos x\ dx &= x^2 \sin x - 2 \left(-x \cos x + \int \cos x\ dx \right) \\
&= x^2 \sin x - 2\left(-x \cos x + \sin x\right) + C \\
&= x^2 \sin x + 2x \cos x - 2 \sin x + C \\
&= (x^2 - 2) \sin x + 2x \cos x + C.
\end{aligned}$$